Multivariate Regression Generalized Likelihood Ratio Tests for FMRI Activation

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Abstract

In neuroscience, an important research question to be investigated, is whether a region or regions of the brain are being activated when a subject is presented a stimulus. A few methods are in use to address this question but they do not jointly take into account the spatial relationship among the set of voxels under consideration. Multivariate regression can determine whether the set of voxels in one, or several re-

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, i, j y_{ji},

$$y_{ji} \quad \beta_{0j} + \beta_{1j}x_{1i} + \cdots + \beta_{qj}x_{qi} \quad + \quad {}_{ji} \qquad (\;$$

$$\begin{aligned} y_{ji} &\quad (\;, x_{1i}, \ldots, x_{qi} \quad \begin{pmatrix} \beta_{0j} \\ \\ \beta_{qj} \end{pmatrix} \quad + \quad {}_{ji} \qquad (\; \\ y_{ji} &\quad x_i' \quad \beta_j \quad + \quad {}_{ji} \\ \times &\quad \times (q + \quad (q + \quad \times \quad \times \end{aligned}$$

i , ..., n j , ..., p

, j n ,

$$\begin{aligned} \begin{pmatrix} y_{j1} \\ \\ y_{jn} \end{pmatrix} &\quad \begin{pmatrix} x_1' \\ \\ x_n' \end{pmatrix} \quad \begin{pmatrix} \beta_{0j} \\ \\ \beta_{qj} \end{pmatrix} \quad + \quad \begin{pmatrix} {}_{j1} \\ \\ {}_{jn} \end{pmatrix} \qquad (\;3 \\ Y_j &\quad X \quad \beta_j \quad + \quad E_j. \\ n \times &\quad n \times (q + \quad (q + \quad \times \quad n \times \end{aligned}$$

${}_{ji}$ σ_j^2

$$p(Y_j|\beta_j, \sigma_j^2, X \quad (\;\pi \;\; ^{-\frac{n}{2}} (\sigma_j^2 \;\; ^{-\frac{n}{2}} e^{-\frac{(\;\;{}_j - X\beta_j)'(\;\;{}_j - X\beta_j)}{2\sigma_j^2}}. \qquad (\;$$

] $\hat{\beta}_j$ j

$$\hat{\beta}_j \quad (X'X \;\; ^{-1} X'Y_j, \qquad (\;5$$

$\hat{\beta}_j$ St t ,

$$\hat{\beta}_j \sim t\left(n - q - \;\;, \beta_j, (n - q - \;\;\; ^{-1} g_j (X'X \;\; ^{-1}\right), \qquad (\;6$$

g_j $(Y_j - X\beta_j)'(Y_j - X\beta_j)$ W_{kk} kk^{th} W $(X'X)^{-1}$ $\hat{\sigma}_j^2$ g_j/n σ_j^2

$$H_0: C_j\beta_j = \gamma_j \quad vs \quad H_1: C_j\beta_j \neq \gamma_j \qquad (2.3)$$
$$\sigma_j^2 > 0 \qquad\qquad \sigma_j^2 > 0$$

C_j $r \times (q+1)$ γ_j $r \times 1$

$$F = \frac{(C\hat{\beta}_j - \gamma_j)'[C(X'X)^{-1}C']^{-1}(C\hat{\beta}_j - \gamma_j)}{rg_j/(n-q-1)} \qquad (2.4)$$

r $n-q-1$ $C_j\beta_j = \gamma_j$

$H_0: \beta_{kj} = \gamma_{kj}$, $C = (\ldots, 0, 1, 0, \ldots)$ $(q+1)$ k^{th}

$$t_{kj} = \frac{\hat{\beta}_{kj} - \gamma_{kj}}{[W_{kk}g_j/(n-q-1)]^{\frac{1}{2}}} \qquad (2.5)$$

$$F_{kj} = \frac{(\hat{\beta}_{kj} - \gamma_{kj})^2}{W_{kk}g_j/(n-q-1)} \qquad (2.6)$$

t $n-q-1$ $n-q-1$ g_j $(X'X)$ $n > q+1$

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i, p y_i,

$$\begin{pmatrix} y_{1i} \\ \vdots \\ y_{pi} \end{pmatrix} = \begin{pmatrix} \beta_{01} + \beta_{11}x_{1i} + \cdots + \beta_{q1}x_{qi} \\ \vdots \\ \beta_{0p} + \beta_{1p}x_{1i} + \cdots + \beta_{qp}x_{qi} \end{pmatrix} + \begin{pmatrix} \epsilon_{1i} \\ \vdots \\ \epsilon_{pi} \end{pmatrix} \qquad (3.1)$$

$$
\begin{array}{ccccccc}
(y_{1i},\ldots,y_{pi} & & (\;, x_{1i},\ldots,x_{qi} & \begin{pmatrix} \beta_{01} & \beta_{02} & \ldots & \beta_{0p} \\ \\ \beta_{q1} & \beta_{q2} & \ldots & \beta_{qp} \end{pmatrix} & + & (\;_{1i},\ldots, \;_{pi} \\
y_i' & & x_i' & (B_0, B_1, \ldots, B_q\;' & + & \;_i' \\
y_i' & & x_i' & B' & + & \;_i' \\
\times p & & \times (q+ & (q+ \quad \times p & & \times p
\end{array}
\quad (3
$$

$i \quad ,\ldots,n$

$, \quad p \quad n \quad ,$

$$
\begin{array}{ccccccc}
\begin{pmatrix} y_1' \\ \\ y_n' \end{pmatrix} & & \begin{pmatrix} x_1' \\ \\ x_n' \end{pmatrix} & \begin{pmatrix} \beta_{01} & \beta_{02} & \ldots & \beta_{0p} \\ \\ \beta_{q1} & \beta_{q2} & \ldots & \beta_{qp} \end{pmatrix} & + & \begin{pmatrix} \;_1' \\ \\ \;_n' \end{pmatrix} \\
Y & & X & B' & + & E. \\
n \times p & & n \times (q+ & (q+ \quad \times p & & n \times p
\end{array}
\quad (3\ 3
$$

p, Y, i^{th}, p, i, Y, j^{th}, n, j

$_i$, p, $p \times p$, Σ, $\hat{B}'$

$$\hat{B}' \quad (X'X \;^{-1}X'Y, \qquad (3$$

$\hat{B}$ St t ,

$$\hat{B} \sim t\left(n-q- \;, B, \; (n-q- \quad (X'X\;]^{-1}, G\right), \qquad (3\ 5$$

$\hat{B}_k$ St t ,

$$\hat{B}_k \sim t\left(n-q-p, B_k, (n-q-p\;^{-1}W_{kk}G\right), \qquad (3\ 6$$

$\hat{\beta}_j$ St t ,

$$\hat{\beta}_j \sim t\left(n-q-p, \beta_j, (n-q-p\;^{-1}g_j(X'X\;^{-1}\right), \qquad (3\ 7$$

$\hat{\beta}_{kj}$ $\hat{B}_{jk}$ St t ,

$$\hat{\beta}_{kj} \sim t$$

4 SIMULATION

O F 3.73

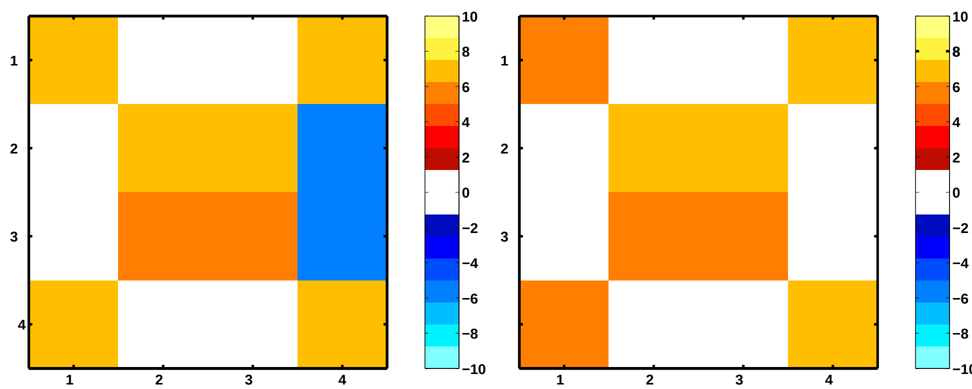
$F_{10^{-6},16,128-2-16}$. 6

G F O

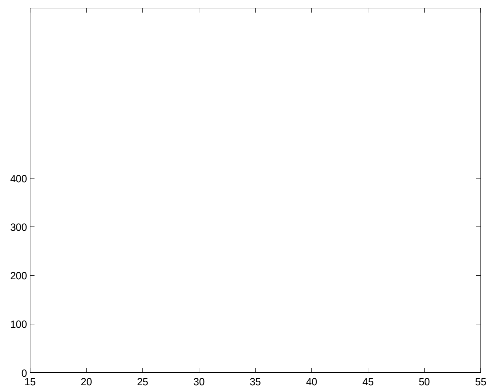
O

ff

S ,



[illegible]


0
100
200
300
400
15
20
25
30
35
40
45
50
55

A.2 Multivariate Likelihood Ratio

S ... , ... $p(Y|B,\Sigma,X$ ... $(B,\Sigma$ ... $(B,\Sigma$ ... $(\hat{B},\hat{\Sigma}$ ...

$$\lambda \quad \frac{p(Y|B,\Sigma,X}{p(Y|\hat{B},\hat{\Sigma},X} \qquad (6$$

$$\frac{(\pi \; ^{-\frac{np}{2}}|\Sigma|^{-\frac{n}{2}}e^{-\frac{1}{2}tr\tilde{\Sigma}^{-1}(Y-X\tilde{B}')(Y-X\tilde{B}')'}}{(\pi \; ^{-\frac{np}{2}}|\hat{\Sigma}|^{-\frac{n}{2}}e^{-\frac{1}{2}tr\hat{\Sigma}^{-1}(Y-X\hat{B}')(Y-X\hat{B}')'}} \qquad (7$$

$$\lambda^{-\frac{2}{n}} \quad \frac{|(Y - XB' \; (Y - XB' \; '|}{|(Y - X\hat{B}' \; (Y - X\hat{B}' \; '|} \qquad (8$$

$$|G + (\hat{B} - B \; (X'X \; (\hat{B} - B \; '|/|G| \qquad ($$

... C (, I , Λ $\lambda^{\frac{2}{n}}$ ... W ,

[illegible]

B'	[illegible] 3 5 6 7 8 [illegible] 3 5 6
	[illegible] 7 3 5 [illegible] 5 6 8
	5 [illegible] 6 8 3 7 3 5 6 [illegible] 5
[illegible]	5 5 3 5 5 3 3 5 5 3 5 5
$\hat{B}'$	[illegible] 3 5 6 7 8 [illegible] 3 5 6
	3 7 [illegible] 8 6 8 [illegible] [illegible] [illegible] 3 5 5 [illegible] 5
	5 [illegible] 6 8 3 7 3 5 6 [illegible] 5
[illegible]	5 8 5 5 [illegible] 6 5 6 3 7 [illegible] 8 3 7 7 5

[illegible] 3 [illegible] [illegible]

Σ/σ^2		[illegible]	3		5	6	7	8				[illegible]	3		5	6
		ρ			ρ											
[illegible]	ρ		ρ			ρ										
3		ρ		ρ			ρ									
			ρ					ρ								
5	ρ					ρ			ρ							
6		ρ			ρ		ρ			ρ						
7			ρ			ρ		ρ			ρ					
8				ρ			ρ					ρ				
					ρ					ρ			ρ			
						ρ			ρ		ρ			ρ		
							ρ			ρ		ρ			ρ	
[illegible]								ρ			ρ					ρ
3									ρ					ρ		
										ρ			ρ		ρ	
5											ρ			ρ		ρ
6												ρ				

$\hat{\Sigma}$	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
1	71.9	23.4	12.3	-7.6	11.0	-7.0	8.4	-2.6	3.9	-2.0	3.8	-0.5	-16.1	-9.9	4.4	-3.7
2	23.4	59.9	7.8	-5.6	-7.4	14.5	1.1	1.1	8.3	4.4	-0.9	2.5	-4.2	-7.5	-4.4	-7.3
3	12.3	7.8	66.7	19.6	4.6	0.2	13.1	3.2	10.7	0.0	1.6	10.3	7.7	-2.2	1.0	-6.6
4	-7.6	-5.6	19.6	57.1	0.8	1.7	-1.0	15.4	-1.8	-4.1	3.1	17.3	2.4	-6.9	-6.0	-6.8
5	11.0	-7.4	4.6	0.8	55.9	18.6	3.3	-2.6	19.5	5.4	5.4	-10.7	-0.2	-5.6	2.0	-3.9
6	-7.0	14.5	0.2	1.7	18.6	57.3	16.9	2.7	11.7	17.5	1.7	-5.2	-0.9	4.7	-2.4	3.8
7	8.4	1.1	13.1	-1.0	3.3	16.9	55.3	23.3	7.0	2.3	19.5	4.7	-5.7	-3.1	3.5	4.2
8	-2.6	1.1	3.2	15.4	-2.6	2.7	23.3	57.8	-1.9	3.3	10.8	18.7	2.8	-3.0	9.2	3.5
9	3.9	8.3	10.7	-1.8	19.5	11.7	7.0	-1.9	76.5	28.9	1.8	-10.4	13.3	-4.2	-3.3	-3.3
10	-2.0	4.4	0.0	-4.1	5.4	17.5	2.3	3.3	28.9	71.5	25.0	-2.7	0.9	11.4	4.4	-3.5
11	3.8	-0.9	1.6	3.1	5.4	1.7	19.5	10.8	1.8	25.0	73.0	16.1	4.5	1.8	17.7	-0.6
12	-0.5	2.5	10.3	17.3	-10.7	-5.2	4.7	18.7	-10.4	-2.7	16.1	71.7	-6.1	1.1	-1.2	14.0
13	-16.1	-4.2	7.7	2.4	-0.2	-0.9	-5.7	2.8	13.3	0.9	4.5	-6.1	67.3	14.4	1.7	-0.9
14	-9.9	-7.5	-2.2	-6.9	-5.6	4.7	-3.1	-3.0	-4.2	11.4	1.8	1.1	14.4	59.2	15.9	5.6
15	4.4	-4.4	1.0	-6.0	2.0	-2.4	3.5	9.2	-3.3	4.4	17.7	-1.2	1.7	15.9	58.9	22.5
16	-3.7	-7.3	-6.6	-6.8	-3.9	3.8	4.2	3.5	-3.3	-3.5	-0.6	14.0	-0.9	5.6	22.5	60.9

References